# Tri-District
# Mathematics Curriculum
## 2010
## Geometry


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# GEOMETRY CURRICULUM
# UNIT 1: GEOMETRIC PROOF & REASONING

## STANDARDS

## Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## *Geometry*

### Congruence G-CO

Experiment with transformations in the plane

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Prove geometric theorems

9. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

Make geometric constructions

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

### Expressing Geometric Properties with Equations G-GPE

Use coordinates to prove simple geometric theorems algebraically

4. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).
5. Prove the slope criteria for parallel and perpendicular lines and use

them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

**Modeling with Geometry** **G-MG**

Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

## BIG IDEAS/COMMON THREADS

All students will understand the meaning of numbers, how they may be represented and the relationships among them. They will perform computations and acquire knowledge of the physical world from the point of view of quantitative relationships.

## ENDURING UNDERSTANDINGS

Reasoning and/or proof can be used to verify or refute conjectures or theorems in geometry.

## ESSENTIAL QUESTIONS

PRIMARY: How do you come to a valid conclusion using inductive and/or deductive reasoning?

SECONDARY: Why are points, lines, and planes considered the foundation of geometry?

## UNIT ASSESSMENT

(see attached)

## LESSON OBJECTIVES

Students will...

- understand that geometric diagrams can be used to test conjectures and identify logical errors in fallacious proofs.
- know and use (in reasoning and problem solving) definitions of angles, polygons, parallel, and perpendicular lines, rigid motions, parallelograms and rectangles.
- prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are

congruent; two lines parallel to a third are parallel to each other; points on a perpendicular bisector of a segment are exactly those equidistant from the segment's endpoints.
- make formal geometric constructions with a variety of tools and methods.
- understand that two lines with well-defined slopes are perpendicular if and only if the product of their slopes is equal to –1.
- use the slope criteria for parallel and perpendicular lines to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
- recognize conditional statements to introduce basic postulates.
- understand basic terms and postulates of geometry: segment/angle addition postulates, linear pair postulate, midpoints, angle bisectors, right angle congruence.
- understand the difference between deductive and inductive reasoning.
- connect reasoning in Algebra and Geometry.

## MODULE SKILLS

Students will be able to…

- identify basic elements in geometry.
- find the measures of segments and angles.
- write converse, inverse, contrapositive, or biconditional statements, given a conditional statement.
- apply the Law of Detachment and Syllogism to mathematical and real world situations.
- use inductive reasoning to make conjectures.
- use deductive and inductive reasoning to prove theorems.
- use coordinates to prove simple geometric theorems algebraically.
- apply properties of equality and congruence.
- state and use the distance and midpoint formulas.
- find the point on a directed line segment between two given points that partitions the segment in a given ratio.
- prove statements using the basic definitions, postulates, and theorems of geometry.
- identify the angle pairs created from parallel lines cut by a transversal.
- prove lines are parallel using angle relationships.
- use properties of parallel and perpendicular lines to solve problems.
- apply algebraic skills to problems involving the terms and postulates throughout the unit.
- apply basic terms, postulates, and theorems to solve real world problems.

## RESOURCES

Supplies: Calculators, protractors, rulers, and grid paper
Texts: Prentice Hall Geometry, McDougal Littell Geometry
Technology: Geometer's Sketchpad Software, ExamView Test Generator

# GEOMETRY CURRICULUM
# UNIT 2: TRIANGLES

## STANDARDS

## Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## *Geometry*

**Congruence** **G-CO**

Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems

10. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

Make geometric constructions

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

**Similarity, Right Triangles, and Trigonometry** **G-SRT**

Understand similarity in terms of similarity transformations

2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity

4. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

**Expressing Geometric Properties with Equations** **G-GPE**

Use coordinates to prove simple geometric theorems algebraically

4. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).

**Modeling with Geometry** **G-MG**

Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

## BIG IDEAS/COMMON THREADS

All students will understand the meaning of numbers, how they may be represented and the relationships among them. They will perform computations and acquire knowledge of the physical world from the point of view of quantitative relationships.

## ENDURING UNDERSTANDINGS

Triangles are a prevalent shape in real world applications based on their distinctive attributes.

## ESSENTIAL QUESTIONS

PRIMARY: What are the distinctive attributes of triangles that make them so valuable in the real world?

SECONDARY: How are these attributes applied to solve real world problems?

## UNIT ASSESSMENT

(see attached)

## LESSON OBJECTIVES

Students will...

- understand that criteria for triangle congruence are ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent.
- know and use (in reasoning and problem solving) definitions of angles, polygons, parallel, and perpendicular lines, rigid motions, parallelograms and rectangles.
- understand that criteria for triangle congruence (ASA, SAS, and SSS) can be established using rigid motions.
- prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°, base angles of isosceles triangles are congruent, the triangle inequality, the longest side of a triangle faces the angle with the greatest measure and vice-versa, the exterior-angle inequality, and the segment joining midpoints of two sides of a triangle parallel to the third side and half the length.
- understand that the assumed properties of dilations can be used to establish the AA, SAS, and SSS criteria for similarity of triangles.
- understand that a line parallel to one side of a triangle divides the other two proportionally, and conversely.
- use triangle similarity criteria to solve problems and to prove relationships in geometric figures. Include a proof of the Pythagorean Theorem using triangle similarity.
- understand that special segments can be drawn within a triangle, each with distinct properties.
- understand the concept of similarity and how that concept can be utilized to solve real world problems.

## MODULE SKILLS

Students will be able to...

- use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

- find triangle angle measures using triangle-angle-sum and exterior angle theorems.
- prove triangles congruent using SSS, SAS, ASA, AAS, and HL.
- prove corresponding parts of triangles congruent.
- use and apply properties of isosceles and equilateral triangles.
- use properties of midsegments to solve problems.
- make formal geometric constructions with a variety of tools and methods
- identify and use properties of angle bisectors, perpendicular bisectors, medians and altitudes in triangles.
- use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- use inequalities involving angles and sides of triangles.
- prove triangles similar using AA, SSS, and SAS and apply the properties of similar triangles.
- use the similarity ratio to find side lengths, perimeter, and area of similar triangles.
- find and use the geometric mean in solving similar right triangles.
- use the side-splitter and triangle-angle-bisector theorems in solving similar right triangles.

## RESOURCES

Supplies: Calculators, protractors, rulers, and grid paper
Texts: Prentice Hall Geometry, McDougal Littell Geometry
Technology: Geometer's Sketchpad Software, ExamView Test Generator

# GEOMETRY CURRICULUM
# UNIT 3: RIGHT TRIANGLE TRIGONOMETRY

## STANDARDS

### Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### *Geometry*

**Similarity, Right Triangles, and Trigonometry** **G-SRT**

Define trigonometric ratios and solve problems involving right triangles

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

**Modeling with Geometry** **G-MG**

Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

## BIG IDEAS/COMMON THREADS

All students will understand the meaning of numbers, how they may be represented and the relationships among them. They will perform computations and acquire knowledge of the physical world from the point of view of quantitative relationships.

## ENDURING UNDERSTANDINGS

Trigonometry examines relationships between the sides and angles of triangles. Trigonometric functions are one way to describe those relationships.

## ESSENTIAL QUESTIONS

PRIMARY: How are trigonometry and its functions applied to solve for parts of triangles?
SECONDARY: Why is trigonometry so valuable in the real world?

## UNIT ASSESSMENT

(see attached)

## LESSON OBJECTIVES

Students will...

- understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of sine, cosine, and tangent.
- use sine, cosine, tangent, and the Pythagorean Theorem to solve right triangles in applied problems.
- find areas of polygons by dissecting them into triangles.
- understand the relationship between side lengths of special right triangles, i.e. 30-60-90 and 45-45-90.
- understand the relationship between similar triangles and trigonometric ratios.
- understand when and how to apply trigonometric ratios to solve right triangles.
- understand how trigonometry is essential to solve for area of regular polygons.
- understand how trigonometry is used to solve for angles of elevation and depression.
- understand how trigonometry is used to solve for vector length and direction.

## MODULE SKILLS

Students will be able to...

- solve for all parts of special right triangles.
- solve for all parts of right triangles.
- explain and use the relationship between the sine and cosine of complementary angles.
- identify and solve angles of elevation and depression in application problems.
- find the area of regular polygons using trigonometry.
- solve for vector length and direction in application problems.

## RESOURCES

Supplies: Calculators, protractors, rulers, and grid paper
Texts: Prentice Hall Geometry, McDougal Littell Geometry
Technology: Geometer's Sketchpad Software, ExamView Test Generator

# GEOMETRY CURRICULUM
# UNIT 4: POLYGONS

## STANDARDS

## Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## *Geometry*

**Congruence** **G-CO**

Experiment with transformations in the plane

2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

Prove geometric theorems

11. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Make geometric constructions

12. Make formal geometric constructions with a variety of tools and

methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

**Similarity, Right Triangles, and Trigonometry** **G-SRT**

Understand similarity in terms of similarity transformations

1. Verify experimentally the properties of dilations given by a center and a scale factor:
   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

**Circles** **G-C**

Understand and apply theorems about circles

3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

**Expressing Geometric Properties with Equations** **G-GPE**

Use coordinates to prove simple geometric theorems algebraically

4. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

**Modeling with Geometry** **G-MG**

Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

## BIG IDEAS/COMMON THREADS

All students will understand the meaning of numbers, how they may be represented and the relationships among them. They will perform computations and acquire knowledge of the physical world from the point of view of quantitative relationships.

## ENDURING UNDERSTANDINGS

Polygons possess distinctive properties which form the foundation of our three-dimensional world.

## ESSENTIAL QUESTIONS

PRIMARY: What are the distinct properties of various polygons?
SECONDARY: How are polygon properties applied in the world around us?

## UNIT ASSESSMENT

(see attached)

## LESSON OBJECTIVES

Students will...

- understand that two geometric figures are congruent if there is a sequence of rigid motions (rotations, reflections, translations) that carries one onto the other. This is the principle of superposition.
- know and use (in reasoning and problem solving) definitions of angles, polygons, parallel, and perpendicular lines, rigid motions, parallelograms and rectangles.
- use and prove properties of and relationships among special quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid and kite.
- characterize parallelograms in terms of equality of opposite sides, in terms of equality of opposite angles, and in terms of bisection of diagonals; characterize rectangles as parallelograms with equal diagonals.
- use two-dimensional representations to transform figures and to predict the effect of translations, rotations, and reflections.
- use two-dimensional representations to transform figures and to predict the effect of dilations.
- use coordinates to compute perimeters of polygons and areas for triangles and rectangles, e.g. using the distance formula.
- use coordinates to prove simple geometric theorems algebraically.
- understand that the area of a decomposed figure is the sum of the areas of its components and is independent of the choice of dissection.
- recognize that various polygons possess specific properties.

- determine properties of polygons using inductive and deductive reasoning.
- understand that area is a two dimensional measurement.
- understand proportionality properties of similar polygons.
- verify experimentally the properties of dilations given by a center and a scale factor:
    - A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
    - The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
- Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- recognize that polygonal shapes are present in both the natural and man-made worlds.

## MODULE SKILLS

Students will be able to...

- classify quadrilaterals according to their properties.
- solve for angle measures and segment lengths associated with parallelograms, trapezoids, and kites using their properties.
- describe which polygons are concave or convex.
- use inductive reasoning to derive interior and exterior angle measure theorems.
- use inductive reasoning to derive the formula for the number of diagonals in a polygon.
- make formal geometric constructions with a variety of tools and methods.
- utilize algebra to find the interior and exterior angle measures of polygons.
- determine which polygons tessellate and why.
- prove a quadrilateral is a parallelogram, trapezoid, or kite using logic and coordinate geometry.
- determine whether a parallelogram is a rectangle, rhombus, both, or neither.
- find the area of parallelograms, trapezoids, kites and composite figures.
- use the similarity ratio to find side lengths, perimeter, and area of similar figures.
- perform transformations including translations, reflections, rotations, and dilations on polygons within the coordinate plane.

## RESOURCES

Supplies: Calculators, protractors, rulers, and grid paper
Texts: Prentice Hall Geometry, McDougal Littell Geometry
Technology: Geometer's Sketchpad Software, ExamView Test Generator

# GEOMETRY CURRICULUM
# UNIT 5: CIRCLES

## STANDARDS

## Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## *Geometry*

**Congruence** **G-CO**

Make geometric constructions

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

**Circles** **G-C**

Understand and apply theorems about circles

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Find arc lengths and areas of sectors of circles

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

**Expressing Geometric Properties with Equations** **G-GPE**

Translate between the geometric description and the equation for a conic section

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Use coordinates to prove simple geometric theorems algebraically

4. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).

**Modeling with Geometry** **G-MG**

Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

## BIG IDEAS/COMMON THREADS

All students will understand the meaning of numbers, how they may be represented and the relationships among them. They will perform computations and acquire knowledge of the physical world from the point of view of quantitative relationships.

## ENDURING UNDERSTANDINGS

Relationships exist among angles, segment lengths, circumference, and area of circles.

## ESSENTIAL QUESTIONS

PRIMARY: What are the relationships created by the intersection of lines and/or segments with a circle?

SECONDARY: How are the relationships created by the intersection of lines and/or segments with a circle applied?

## UNIT ASSESSMENT

(see attached)

## LESSON OBJECTIVES

Students will...

- identify and define radius, diameter, chord, tangent, secant, and circumference.

- identify and describe relationships among angles, radii, and chords. Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
- determine the arc lengths and the areas of sectors of circles, using proportions.
- prove theorems about circles, and use these theorems to solve problems involving:
    - Symmetries of a circle
    - Similarity of a circle to any other
    - Tangent line, perpendicularity to a radius
    - Inscribed angles in a circle, relationship to central angles, and equality of inscribed angles
    - Properties of chords, tangents, and secants as an application of triangle similarity
- understand that the equation of a circle can be found using its definition and the Pythagorean Theorem.
- understand that transforming the graph of an equation by reflecting in the axes, translating parallel to the axes, or applying a dilation in one of the coordinate directions corresponds to substitutions in the equation.
- decide whether a point with given coordinates lies on a circle defined by a given equation.
- complete the square to find the center and radius of a circle given by an equation.
- identify major arcs, minor arcs and semi-circles.
- understand the angle and segment relationships created by the intersection of tangents, secants and chords with a circle.
- understand the relationship between circle/sector area and circumference/arc length.
- understand geometric probability.
- understand the connection between circles, their graphs and their equations.

## MODULE SKILLS

Students will be able to...

- use theorems about circles, and use these theorems to solve problems involving:
    - symmetries of a circle
    - Similarity of a circle to any other
    - Tangent line, perpendicularity to a radius
    - Inscribed angles in a circle, relationship to central angles, and equality of inscribed angles
    - Properties of chords, tangents, and secants as an application of triangle similarity.

- differentiate between major arcs, minor arcs and semi-circles.
- find measures of central angles, inscribed angles and angles formed by tangents, secants and chords.
- make formal geometric constructions with a variety of tools and methods.
  construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
- solve for area, circumference, arc length, arc measure and sector area.
- use circle relationships in deductive proof.
- use segment and area models to find the probabilities of events.
- find lengths of tangent segments, secant segments and chords.
- write the equation of a circle based on its graph.
- graph a circle based on its equation.
- identify a locus.
- draw and describe a locus.

## RESOURCES

Supplies: Calculators, protractors, rulers, and grid paper
Texts: Prentice Hall Geometry, McDougal Littell Geometry
Technology: Geometer's Sketchpad Software, ExamView Test Generator

# GEOMETRY CURRICULUM
# UNIT 6: SOLIDS

## STANDARDS

## Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## *Geometry*

**Geometric Measurement and Dimension** **G-GMD**

Explain volume formulas and use them to solve problems

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Visualize relationships between two-dimensional and three- dimensional objects

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

**Modeling with Geometry** **G-MG**

Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

## BIG IDEAS/COMMON THREADS

All students will understand the meaning of numbers, how they may be represented and the relationships among them. They will perform

computations and acquire knowledge of the physical world from the point of view of quantitative relationships.

## ENDURING UNDERSTANDINGS

Solid geometry studies the surface of a three-dimensional figure and the space it encloses.

## ESSENTIAL QUESTIONS

PRIMARY: How are two-dimensional relationships connected to the properties of three-dimensional figures?
SECONDARY: How are two-dimensional measurement concepts used to calculate measures of three-dimensional figures?

## UNIT ASSESSMENT

(see attached)

## LESSON OBJECTIVES

Students will...

- explain why the volume of a cylinder is the area of the base times the height, using informal arguments.
- for a pyramid or a cone, give a heuristic argument to show why its volume is one-third of its height times the area of its base.
- apply formulas and solve problems involving volume and surface area of right prisms, right circular cylinders, right pyramids, cones, spheres and composite figures.
- use geometric shapes, their measures and their properties to describe objects.
- understand that nets can be used to represent three-dimensional figures.
- understand the relationship between a three-dimensional figure and its surface area.
- understand the relationship between a three-dimensional figure and its volume.
- understand the relationship between volumes and surface areas of similar three-dimensional figures.

## MODULE SKILLS

Students will be able to...

- analyze the nets of three-dimensional figures.
- estimate, calculate, and/or compare the surface area of three-dimensional figures and their parts.
- estimate, calculate, and/or compare the volume of three-dimensional figures and their parts.

- utilize algebra and trigonometry to calculate the surface area and volume of three-dimensional figures.
- calculate the volume and surface area of similar three-dimensional figures.
- use the similarity ratio to find side surface area and volume of similar figures.

## RESOURCES

Supplies: Calculators, protractors, rulers, and grid paper
Texts: Prentice Hall Geometry, McDougal Littell Geometry
Technology: Geometer's Sketchpad Software, ExamView Test Generator

# GEOMETRY CURRICULUM
# UNIT 7: PROBABILITY & STATISTICS

## STANDARDS

## Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## *Statistics and Probability*

**Interpreting Categorical and Quantitative Data** **S-ID**

Summarize, represent, and interpret data on a single count or measurement variable

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

**Making Inferences and Justifying Conclusions** **S-IC**

Understand and evaluate random processes underlying statistical experiments

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

**Conditional Probability and The Rules of Probability** **S-CP**

Understand independence and conditional probability and use them to interpret data

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).

2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Use the rules of probability to compute probabilities of compound events in a uniform probability model

6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

## BIG IDEAS/COMMON THREADS

All students will understand the meaning of numbers, how they may be represented and the relationships among them. They will perform computations and acquire knowledge of the physical world from the point of view of quantitative relationships.

## ENDURING UNDERSTANDINGS

Statistics and probability provide tools for describing variability in data and for making informed decisions that take it into account.

## ESSENTIAL QUESTIONS

PRIMARY: How are the laws of probability used to predict outcomes in the real world?

SECONDARY: How is statistics used to analyze data in real world situations?

## UNIT ASSESSMENT
(see attached)

## LESSON OBJECTIVES
Students will...

- understand and evaluate random processes underlying statistical experiments.
- understand that in a probability model, individual outcomes have probabilities that sum to 1. When outcomes are categorized, the probability of a given type of outcome is the sum of the probabilities of all the individual outcomes of that type.
- understand that uniform probability models are useful models for processes such as
    - (i) the selection of a person from a population;
    - (ii) the selection of a number in a lottery;
    - (iii) any physical situation in which symmetry suggests that different individual outcomes are equally likely.
- understand that two different empirical probability models for the same process will rarely assign exactly the same probability to a given type of outcome. But if the data sets are large and the methods used to collect the data for the two data sets are consistent, the agreement between the models is likely to be reasonably good.
- understand that a (theoretical) uniform probability model may be judged by comparing it to an empirical probability model for the same process. If the theoretical assumptions are appropriate and the data set is large, then the two models should agree approximately. If the agreement is not good, then it may be necessary to modify the assumptions underlying the theoretical model or look for factors that might have affected the data used to create the empirical model.
- understand that to describe a pair of random processes (such as tossing a coin and rolling a number cube), or one random process repeated twice (such as randomly selecting a student in the class on two different days), two probability models can be combined into a single model.
    - the sample space for the combined model is formed by listing all possible ordered pairs that combine an individual outcome from the first model with an individual outcome from the second. Each ordered pair is an individual outcome in the combined model.
    - the total number of individual outcomes (ordered pairs) in the combined model is the product of the number of individual outcomes in each of the two original models.
- understand that when two probability models are combined independently, the probability that one type of outcome in the first model occurs together with another type of outcome in the second model is the product of the two corresponding probabilities in the original models (the Multiplication Rule).

- understand that events are subsets of a sample space; often, events of interest are defined by using characteristics (or categories) of the sample points, or as unions, intersections, or complements thereof (“and,” “or,” “not”). A sample point may belong to several events (categories).
- understand that if A and B are two events, then in a uniform model the conditional probability of A given B, denoted by P(A|B), is the fraction of B‘s sample points that also lie in A.
- understand that the laws of probability allow one to use known probabilities to determine other probabilities of interest.

## MODULE SKILLS

Students will be able to…

- represent data with plots on the real number line (dot plots, histograms, and box plots).
- use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
- interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
- use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
- generate data by sampling, repeated experimental trials, and simulations. Record and appropriately label such data, and use them to construct an empirical probability model. Compute probabilities in such models.
- use a uniform probability model to compute probabilities for a process involving uncertainty, including the random selection of a person from a population and physical situations where symmetry suggests that different individual outcomes are equally likely.
    - list the individual outcomes to create a sample space.
    - label the individual outcomes in the sample space to reflect important characteristics or quantities associated with them.
    - determine probabilities of individual outcomes, and determine the probability of a type or category of outcome as the fraction of individual outcomes it includes.
- compare probabilities from a theoretical model to probabilities from a corresponding empirical model for the same situation. If the agreement is not good, explain possible sources of the discrepancies.
- combine two uniform models independently to compute probabilities for a pair of random processes (e.g., flipping a coin twice, selecting one person from each of two classes).

    - Use organized lists, tables and tree diagrams to represent the combined sample space.
    - Determine probabilities of ordered pairs in the combined model, and determine the probability of a particular type or category of outcomes in the combined model, as the fraction of ordered pairs corresponding to it.
- for two independently combined uniform models, use the Multiplication Rule to determine probabilities.
- compute probabilities by constructing and analyzing sample spaces, representing them by tree diagrams, systematic lists, and Venn diagrams.
- use the laws of probability to compute probabilities.
- apply concepts such as intersections, unions and complements of events, and conditional probability and independence to define or analyze events, calculate probabilities and solve problems.
- construct and interpret two-way tables to show probabilities when two characteristics (or categories) are associated with each sample point.
- use a two-way table to determine conditional probabilities.
- recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
- use permutations and combinations to compute probabilities of compound events and solve problems.

## RESOURCES

Supplies: Calculators, protractors, rulers, and grid paper
Texts: Prentice Hall Geometry, McDougal Littell Geometry, Supplemental Statistics & Probability Books
Technology: Geometer's Sketchpad Software, ExamView Test Generator