Tri-District
Mathematics Curriculum
2009
Grade Six

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STATE STANDARD

4.1.6A.1. Use real-life experiences, physical materials, and technology to construct meanings for numbers.
   • All integers
   • All fractions as part of a whole, as subset of a set, as a location on a number line, and as divisions of whole numbers
   • All decimals

4.1.6A.7. Develop and apply number theory concepts in problem solving situations.
   • Primes, factors, multiples
   • Common multiples, common factors

4.1.6A.8. Compare and order numbers.

4.1.6B.5. Find squares and cubes of whole numbers.

4.1.6A.1. Recognize, describe, extend, and create patterns involving whole numbers and rational numbers.
   • Descriptions using tables, verbal rules, simple equations, and graphs
   • Formal iterative formulas (e.g., $\text{NEXT} = \text{NOW} \times 3$)
   • Recursive patterns, including Pascal’s Triangle (where each entry is the sum of the entries above it) and the Fibonacci Sequence: $1, 1, 2, 3, 5, 8, \ldots$ (where $\text{NEXT} = \text{NOW} + \text{PREVIOUS}$)

4.3.6C.1. Use patterns, relations, and linear functions to model situations.
   • Using variables to represent unknown quantities
   • Using concrete materials, tables, graphs, verbal rules, algebraic expressions/equations/inequalities

4.3.6D.2. Understand and apply the properties of operations and numbers.
   • Distributive property
   • The product of a number and its reciprocal is 1

4.3.6D.3. Evaluate numerical expressions.

4.4.6C.1. Solve counting problems and justify that all possibilities have been enumerated without duplication.
   • Organized lists, charts, tree diagrams, tables
   • Venn diagrams

4.4.6D.1. Devise strategies for winning simple games (e.g., start with two piles of objects, each of two players in turn removes any number of objects from a single pile, and the person to take the last group of objects wins) and express those strategies as sets of directions.

4.5A.1. Learn mathematics through problem solving, inquiry, and discovery.
4.5A.2. Solve problems that arise in mathematics and in other contexts.
  • Open-ended problems
  • Non-routine problems
  • Problems with multiple solutions
  • Problems that can be solved in several ways
4.5A.3. Select and apply a variety of appropriate problem-solving strategies (e.g., “try a simpler problem” or “make a diagram”) to solve problems.
4.5A.4. Pose problems of various types and levels of difficulty.
4.5A.5. Monitor their progress and reflect on the process of their problem solving activity.
4.5B.1. Use communication to organize and clarify mathematical thinking.
  • Reading and writing
  • Discussion, listening, and questioning
4.5B.2. Communicate mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
4.5B.3. Analyze and evaluate the mathematical thinking and strategies of others.
4.5B.4. Use the language of mathematics to express mathematical ideas precisely.
4.5C.1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
4.5C.2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
4.5C.3. Recognize that mathematics is used in a variety of contexts outside of mathematics.
4.5C.4. Apply mathematics in practical situations and in other disciplines.
4.5C.5. Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
4.5C.6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
4.5D.1. Recognize that mathematical facts, procedures, and claims must be justified.
4.5D.2. Use reasoning to support their mathematical conclusions and problem solutions.
4.5D.3. Select and use various types of reasoning and methods of proof.
4.5D.4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
4.5D.5. Make and investigate mathematical conjectures.
  • Counterexamples as a means of disproving conjectures
  • Verifying conjectures using informal reasoning or proofs.
4.5D.6. Evaluate examples of mathematical reasoning and determine whether they are valid.
4.5E.1. Create and use representations to organize, record, and communicate mathematical ideas.
  • Concrete representations (e.g., base-ten blocks or algebra tiles)
  • Pictorial representations (e.g., diagrams, charts, or tables)
• Symbolic representations (e.g., a formula)
• Graphical representations (e.g., a line graph)

4.5E.2. Select, apply, and translate among mathematical representations to solve problems.
4.5E.3. Use representations to model and interpret physical, social, and mathematical phenomena.
4.5F.1. Use technology to gather, analyze, and communicate mathematical information.
4.5F.2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.
4.5F.3. Use graphing calculators and computer software to investigate properties of functions and their graphs.
4.5F.4. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
4.5F.5. Use computer software to make and verify conjectures about geometric objects.
4.5F.6. Use computer-based laboratory technology for mathematical applications in the sciences.

BIG IDEAS/COMMON THREADS
All students will understand the meaning of numbers, how they may be represented and the relationships among them. They will perform computations and acquire knowledge of the physical world from the point of view of quantitative relationships.

ENDURING UNDERSTANDINGS
Whole numbers have properties that help us to solve problems.

ESSENTIAL QUESTIONS
PRIMARY: What relationships are revealed when numbers are viewed as multiples and broken into factors?
SECONDARY: How do these relationships help me solve problems?

MODULE ASSESSMENT
Unit 1 Assessment

LESSON OBJECTIVES
Students will be able to...
• understand relationships among factors, multiples, divisors, and products.
• recognize and use properties of prime and composite numbers, even and odd numbers, and square numbers.
• use rectangles to represent the factor pairs of numbers.
• develop strategies for finding factors and multiples, least common multiples, and greatest common factors.
• recognize and use the fact that every whole number can be written in exactly one way as a product of prime numbers.
• use factors and multiples to solve problems and to explain some numerical facts of everyday life.
• develop a variety of strategies for solving problems – building models, making lists and tables, drawing diagrams, and solving simpler problems.

MODULE SKILLS
Students will be able to...
• determine the factors, multiples, divisors, and products of a given number.
• classify a number by the sum of the proper factors (i.e. abundant, proper, deficient, or perfect).
• identify the proper factors of a given number.
• identify if a number is prime or composite, even or odd, and/or square.
• identify if the sum or product of two numbers is even or odd, given whether each original number is even or odd.
• represent the factor pairs of a number, both numerically and as rectangles.
• find factors of a number using different strategies, including repeated multiplication and number trees.
• find multiples of a number.
• given two numbers, use different strategies to find the least common multiple (i.e. numerically, using Venn Diagrams, or using prime factorizations).
• given two numbers, use different strategies to find the greatest common factor (i.e. numerically, using Venn Diagrams, or using prime factorizations).
• find the prime factorization of any number (i.e. the Fundamental Theorem of Arithmetic).
• solve simple real-world problems that involve the concepts of factors and multiples.
• use different strategies to solve problems (i.e. building models, making lists and tables, drawing diagrams, and solving simpler problems).

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STATE STANDARD
4.1.6A.1 Use real-life experiences, physical materials, and technology to construct meanings for numbers
- All integers
- All fractions as part of a whole, as subset of a set, as a location on a number line, and as divisions of whole numbers
- All decimals
4.1.6A.2 Recognize the decimal nature of United States currency and compute with money.
4.1.6A.3 Demonstrate a sense of the relative magnitudes of numbers.
4.1.6A.4 Explore the use of ratios and proportions in a variety of situations.
4.1.6A.6 Use whole numbers, fractions, and decimals to represent equivalent forms of the same number.
4.1.6A.8 Compare and order numbers.
4.1.6B.1 Recognize the appropriate use of each arithmetic operation in problem situations.
4.1.6B.2 Construct, use, and explain procedures for performing calculations with fractions and decimals with:
- Pencil-and-paper
- Mental math
- Calculator
4.1.6B.4 Select pencil-and-paper, mental math, or a calculator as the appropriate computational method in a given situation depending on the context and numbers.
4.1.6B.6 Check the reasonableness of results of computations.
4.1.6B.7 Understand and use the various relationships among operations and properties of operations.
4.1.6B.8 Understand and apply the standard algebraic order of operations for the four basic operations, including appropriate use of parentheses.
4.3.6C.1 Use patterns, relations, and linear functions to model situations.
- Using variables to represent unknown quantities
- Using concrete materials, tables, graphs, verbal rules, algebraic expressions/equations/inequalities
4.3.6D.2 Understand and apply the properties of operations and numbers.
- Distributive property
- The product of a number and its reciprocal is 1
4.3.6D.3 Evaluate numerical expressions.
4.4.6D.1 Devise strategies for winning simple games (e.g., start with two piles of objects, each of two players in turn removes any number of objects from a single pile, and the person to take the last group of objects wins) and express those strategies as sets of directions.

4.5A.1 Learn mathematics through problem solving, inquiry, and discovery.

4.5A.2 Solve problems that arise in mathematics and in other contexts.
   - Open-ended problems
   - Non-routine problems
   - Problems with multiple solutions
   - Problems that can be solved in several ways

4.5A.3 Select and apply a variety of appropriate problem-solving strategies (e.g., "try a simpler problem" or "make a diagram") to solve problems.

4.5A.4 Pose problems of various types and levels of difficulty.

4.5A.5 Monitor their progress and reflect on the process of their problem solving activity.

4.5B.1 Use communication to organize and clarify mathematical thinking.
   - Reading and writing
   - Discussion, listening, and questioning

4.5B.2 Communicate mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.

4.5B.3 Analyze and evaluate the mathematical thinking and strategies of others.

4.5B.4 Use the language of mathematics to express mathematical ideas precisely.

4.5C.1 Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).

4.5C.2 Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).

4.5C.3 Recognize that mathematics is used in a variety of contexts outside of mathematics.

4.5C.4 Apply mathematics in practical situations and in other disciplines.

4.5C.5 Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).

4.5C.6 Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

4.5D.1 Recognize that mathematical facts, procedures, and claims must be justified.

4.5D.2 Use reasoning to support their mathematical conclusions and problem solutions.

4.5D.3 Select and use various types of reasoning and methods of proof.

4.5D.4 Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.

4.5D.5 Make and investigate mathematical conjectures.
   - Counterexamples as a means of disproving conjectures.
   - Verifying conjectures using informal reasoning or proofs.
4.5D.6 Evaluate examples of mathematical reasoning and determine whether they are valid.

4.5E.1 Create and use representations to organize, record, and communicate mathematical ideas.
- Concrete representations (e.g., base-ten blocks or algebra tiles)
- Pictorial representations (e.g., diagrams, charts, or tables)
- Symbolic representations (e.g., a formula)
- Graphical representations (e.g., a line graph)

4.5E.2 Select, apply, and translate among mathematical representations to solve problems.

4.5E.3 Use representations to model and interpret physical, social, and mathematical phenomena.

4.5F.1 Use technology to gather, analyze, and communicate mathematical information.

4.5F.2 Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.

4.5F.3 Use graphing calculators and computer software to investigate properties of functions and their graphs.

4.5F.4 Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).

4.5F.5 Use computer software to make and verify conjectures about geometric objects.

4.5F.6 Use computer-based laboratory technology for mathematical applications in the sciences.

**BIG IDEAS/COMMON THREADS**
All students will understand the meaning of numbers, how they may be represented and the relationships among them. They will perform computations and acquire knowledge of the physical world from the point of view of quantitative relationships.

**ENDURING UNDERSTANDINGS**
There are many different and important interpretations of and models for rational numbers which allow us to order and compare quantities.

**ESSENTIAL QUESTIONS**
- PRIMARY: Why is it important to represent rational numbers in a variety of ways?
- SECONDARY: How do these ways help us order and compare quantities?

**MODULE ASSESSMENT**
Unit 2 Assessment
LESSON OBJECTIVES
Students will be able to...

• build an understanding of fractions, decimals, and percents and the relationships between and among these concepts and their representations.
• develop ways to model situations involving fractions, decimals, and percents.
• understand and use equivalent fractions to reason about situations.
• compare and order fractions and decimals.
• move flexibly between fractions, decimals, and percent representations.
• use benchmarks such as 0, ½, 1, and 1 ½ to help estimate the size of a number or sum.
• develop and use benchmarks that relate different forms of representations of rational numbers.
• use physical models and drawings to help reason about a situation.
• look for patterns and describe how to continue the pattern.
• use context to help reason about a situation.
• use estimation to understand a situation.

MODULE SKILLS
Students will be able to...

• identify the numerator and denominator of a fraction.
• explain the relationship between the numerator and denominator of a fraction, specifically the concept of part to whole.
• convert a mixed number to a proper fraction.
• represent a fraction pictorially (i.e. on a grid).
• move flexibly among fraction, decimal, and percent representations (convert between fractions, decimals, and percents).
• model situations involving fractions, decimals, and percents.
• identify equivalent fractions and use them to reason about situations.
• compare and order fractions and decimals.
• divide numerator by denominator to convert to a decimal.
• use benchmarks, such as 0, ½ , 1, 1 ½, and 2, to help estimate the size of number or sum.
• develop and use benchmarks that relate different forms of rational numbers (for example, 50% is the same as ½ or 0.5).
• use context, physical models, drawings, patterns, and estimation to help reason about situations involving rational numbers (i.e. fraction strips, number line models, partition models, grid-area models, and percent bar models).
• describe place value as a fraction with powers of ten as the denominator.

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STATE STANDARD

4.1.6A.1 Use real-life experiences, physical materials, and technology to construct meanings for numbers
- All integers
- All fractions as part of a whole, as subset of a set, as a location on a number line, and as divisions of whole numbers
- All decimals

4.2.6A.1 Understand and apply concepts involving lines and angles.
- Notation for line, ray, angle, line segment
- Properties of parallel, perpendicular, and intersecting lines
- Sum of the measures of the interior angles of a triangle is 180°

4.2.6A.2 Identify, describe, compare, and classify polygons and circles.
- Triangles by angles and sides
- Quadrilaterals, including squares, rectangles, parallelograms, trapezoids, rhombi
- Polygons by number of sides.
- Equilateral, equiangular, regular
- All points equidistant from a given point form a circle

4.2.6A.4 Understand and apply the concepts of congruence and symmetry (line and rotational).

4.2.6B.2 Recognize, identify, and describe geometric relationships and properties as they exist in nature, art, and other real-world settings.

4.2.6C.1 Create geometric shapes with specified properties in the first quadrant on a coordinate grid.

4.2.6D.1 Select and use appropriate units to measure angles, area, surface area, and volume.

4.2.6D.2 Use a scale to find a distance on a map or a length on a scale drawing.

4.2.6E.1 Use a protractor to measure angles.

4.4.6D.2 Analyze vertex-edge graphs and tree diagrams.
- Can a picture or a vertex-edge graph be drawn with a single line? (degree of vertex)
- Can you get from any vertex to any other vertex? (connectedness)

4.4.6D.3 Use vertex-edge graphs to find solutions to practical problems.
- Delivery route that stops at specified sites but involves least travel
- Shortest route from one site on a map to another

4.5A.1 Learn mathematics through problem solving, inquiry, and discovery.

4.5A.2 Solve problems that arise in mathematics and in other contexts.
- Open-ended problems
• Non-routine problems
• Problems with multiple solutions
• Problems that can be solved in several ways

4.5A.3 Select and apply a variety of appropriate problem-solving strategies (e.g., “try a simpler problem” or “make a diagram”) to solve problems.
4.5A.4 Pose problems of various types and levels of difficulty.
4.5A.5 Monitor their progress and reflect on the process of their problem solving activity.

4.5B.1 Use communication to organize and clarify mathematical thinking.
• Reading and writing
• Discussion, listening, and questioning

4.5B.2 Communicate mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
4.5B.3 Analyze and evaluate the mathematical thinking and strategies of others.
4.5B.4 Use the language of mathematics to express mathematical ideas precisely.

4.5C.1 Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
4.5C.2 Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
4.5C.3 Recognize that mathematics is used in a variety of contexts outside of mathematics.
4.5C.4 Apply mathematics in practical situations and in other disciplines.
4.5C.5 Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
4.5C.6 Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

4.5D.1 Recognize that mathematical facts, procedures, and claims must be justified.
4.5D.2 Use reasoning to support their mathematical conclusions and problem solutions.
4.5D.3 Select and use various types of reasoning and methods of proof.
4.5D.4 Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
4.5D.5 Make and investigate mathematical conjectures.
• Counterexamples as a means of disproving conjectures.
• Verifying conjectures using informal reasoning or proofs.
4.5D.6 Evaluate examples of mathematical reasoning and determine whether they are valid.

4.5E.1 Create and use representations to organize, record, and communicate mathematical ideas.
• Concrete representations (e.g., base-ten blocks or algebra tiles)
• Pictorial representations (e.g., diagrams, charts, or tables)
• Symbolic representations (e.g., a formula)
• Graphical representations (e.g., a line graph)
4.5E.2 Select, apply, and translate among mathematical representations to solve problems.
4.5E.3 Use representations to model and interpret physical, social, and mathematical phenomena.
4.5F.1 Use technology to gather, analyze, and communicate mathematical information.
4.5F.2 Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.
4.5F.3 Use graphing calculators and computer software to investigate properties of functions and their graphs.
4.5F.4 Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
4.5F.5 Use computer software to make and verify conjectures about geometric objects.
4.5F.6 Use computer-based laboratory technology for mathematical applications in the sciences.

BIG IDEAS/COMMON THREADS
All students will understand the meaning of numbers, how they may be represented and the relationships among them. They will perform computations and acquire knowledge of the physical world from the point of view of quantitative relationships.

ENDURING UNDERSTANDINGS
Shapes and visual patterns are important features of our world that should be recognized, displayed, analyzed, reasoned, and measured.

ESSENTIAL QUESTIONS
PRIMARY: What are the key properties of polygonal shapes that make them useful and attractive?
SECONDARY: Why would we use different types of polygons?

MODULE ASSESSMENT
Unit 3 Assessment

LESSON OBJECTIVES
Students will be able to...
- identify some important properties of polygons.
- recognize polygonal shapes both in and out of the classroom.
- investigate reflection and rotation symmetries of a shape.
- estimate the measure of any angle using reference to a right angle and other benchmark angles.
- use an angle ruler or protractor to measure an angle.
- explore properties of parallel lines and angles created by lines intersecting parallel lines.
• find patterns that help determine angle sums of polygons.
• determine which polygons fit together to cover a flat surface and understand why they fit together.
• explain the property of triangles that makes them useful as a stable structure for building.
• find that the sum of two side lengths of a triangle is greater than the third side length.
• draw or sketch polygons with certain properties.
• reason about and solve problems involving shapes.

MODULE SKILLS
Students will be able to...
• sort shapes according to some special properties and describe these properties.
• name a polygon given the number of sides and angles.
• identify the vertices and angles in a polygon.
• identify if a given polygon is a regular or irregular polygon.
• identify if a shape has rotation symmetry. If so, identify the center point.
• identify if a shape has reflection symmetry. If so, identify the line of symmetry.
• decide what shapes will tile a surface and what common properties these shapes may have.
• measure angles using an angle ruler or protractor.
• identify pairs of angles formed by two parallel lines cut by a transversal as alternative interior, corresponding, or alternate exterior. State which pairs of angles are congruent and which are supplementary.
• identify vertical angles and state that vertical angles are congruent.
• find the sum of the interior angles of any polygon, given the number of sides of the polygon.
• find the measure of an interior angle of a regular polygon.
• find the sum of the exterior angles of any polygon.
• find the measure of an angle in a polygon given the other angle measures.
• state the possible measures of a side of a triangle, given the measures of the other two sides.
• draw a polygon (if possible) given a written description.
• analyze vertex-edge graphs and tree diagrams.
• use vertex-edge graphs to find solutions to practical problems.

RESOURCES
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STATE STANDARD

4.1.6A.1 Use real-life experiences, physical materials, and technology to construct meanings for numbers
   - All integers
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   - All decimals

4.1.6A.2 Recognize the decimal nature of United States currency and compute with money.

4.1.6A.3 Demonstrate a sense of the relative magnitudes of numbers.

4.1.6A.4 Explore the use of ratios and proportions in a variety of situations.

4.1.6A.6 Use whole numbers, fractions, and decimals to represent equivalent forms of the same number.

4.1.6A.8 Compare and order numbers.

4.1.6B.1 Recognize the appropriate use of each arithmetic operation in problem situations.

4.1.6B.2 Construct, use, and explain procedures for performing calculations with fractions and decimals with:
   - Pencil-and-paper
   - Mental math
   - Calculator

4.1.6B.4 Select pencil-and-paper, mental math, or a calculator as the appropriate computational method in a given situation depending on the context and numbers.

4.1.6B.6 Check the reasonableness of results of computations.

4.1.6B.7 Understand and use the various relationships among operations and properties of operations.

4.1.6B.8 Understand and apply the standard algebraic order of operations for the four basic operations, including appropriate use of parentheses.

4.1.6C.1 Use a variety of strategies for estimating both quantities and the results of computations.

4.1.6C.2 Recognize when an estimate is appropriate, and understand the usefulness of an estimate as distinct from an exact answer.

4.1.6C.3 Determine the reasonableness of an answer by estimating the result of operations.

4.1.6C.4 Determine whether a given estimate is an overestimate or an underestimate.

4.3.6C.1 Use patterns, relations, and linear functions to model situations.
   - Using variables to represent unknown quantities
Using concrete materials, tables, graphs, verbal rules, algebraic expressions/equations/inequalities

**4.3.6D.2** Understand and apply the properties of operations and numbers.
- Distributive property
- The product of a number and its reciprocal is 1

**4.3.6D.3** Evaluate numerical expressions.

**4.5A.1** Learn mathematics through problem solving, inquiry, and discovery.

**4.5A.2** Solve problems that arise in mathematics and in other contexts.
- Open-ended problems
- Non-routine problems
- Problems with multiple solutions
- Problems that can be solved in several ways

**4.5A.3** Select and apply a variety of appropriate problem-solving strategies (e.g., “try a simpler problem” or “make a diagram”) to solve problems.

**4.5A.4** Pose problems of various types and levels of difficulty.

**4.5A.5** Monitor their progress and reflect on the process of their problem solving activity.

**4.5B.1** Use communication to organize and clarify mathematical thinking.
- Reading and writing
- Discussion, listening, and questioning

**4.5B.2** Communicate mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.

**4.5B.3** Analyze and evaluate the mathematical thinking and strategies of others.

**4.5B.4** Use the language of mathematics to express mathematical ideas precisely.

**4.5C.1** Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).

**4.5C.2** Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).

**4.5C.3** Recognize that mathematics is used in a variety of contexts outside of mathematics.

**4.5C.4** Apply mathematics in practical situations and in other disciplines.

**4.5C.5** Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).

**4.5C.6** Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

**4.5D.1** Recognize that mathematical facts, procedures, and claims must be justified.

**4.5D.2** Use reasoning to support their mathematical conclusions and problem solutions.

**4.5D.3** Select and use various types of reasoning and methods of proof.

**4.5D.4** Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.

**4.5D.5** Make and investigate mathematical conjectures.
- Counterexamples as a means of disproving conjectures.
• Verifying conjectures using informal reasoning or proofs.

4.5D.6 Evaluate examples of mathematical reasoning and determine whether they are valid.

4.5E.1 Create and use representations to organize, record, and communicate mathematical ideas.

• Concrete representations (e.g., base-ten blocks or algebra tiles)
• Pictorial representations (e.g., diagrams, charts, or tables)
• Symbolic representations (e.g., a formula)
• Graphical representations (e.g., a line graph)

4.5E.2 Select, apply, and translate among mathematical representations to solve problems.

4.5E.3 Use representations to model and interpret physical, social, and mathematical phenomena.

4.5F.1 Use technology to gather, analyze, and communicate mathematical information.

4.5F.2 Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.

4.5F.3 Use graphing calculators and computer software to investigate properties of functions and their graphs.

4.5F.4 Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).

4.5F.5 Use computer software to make and verify conjectures about geometric objects.

4.5F.6 Use computer-based laboratory technology for mathematical applications in the sciences.

BIG IDEAS/COMMON THREADS
All students will understand the meaning of numbers, how they may be represented and the relationships among them. They will perform computations and acquire knowledge of the physical world from the point of view of quantitative relationships.

ENDURING UNDERSTANDINGS
There are many different and important algorithms and visual models for operations with fractions.

ESSENTIAL QUESTIONS
PRIMARY: Why do we need to perform operations involving fractions?
SECONDARY: What algorithms and visual models can help to perform these operations?

MODULE ASSESSMENT
Unit 4 Assessment
LESSON OBJECTIVES
Students will be able to...
- use benchmarks and other strategies to estimate the reasonableness of results of operations with fractions.
- develop ways to model sums, differences, products, and quotients, including the use of areas, fraction strips, and number lines.
- use estimates and exact solutions to make decisions in problems involving fractions.
- look for rules to generalize patterns in numbers.
- use your knowledge of fractions and equivalence of fractions to develop algorithms for adding, subtracting, multiplying, and dividing fractions.
- recognize when addition, subtraction, multiplication, or division is the appropriate operation to solve a problem.
- write fact families to show the inverse relationship between addition and subtraction, and between multiplication and division.
- solve problem using operations on fractions.

MODULE SKILLS
Students will be able to...
- list fact families.
- use number sentences to express sums and differences involving fractions.
- estimate sums, differences, products, and quotients, involving fractions.
- find an underestimate or overestimate of a sum, difference, product, or quotient, involving fractions.
- draw a visual model of a sum or difference involving fractions.
  - Fraction-strip model
  - Number-line model
- use algorithms to add and subtract fractions.
  - Using equivalent fractions to find a common denominator.
- use visual models to represent a product or quotient involving fractions.
  - Area model
  - Partitioning
- use algorithms to multiply fractions.
  - Multiply numerators & multiply denominators
  - Using the distributive property
- use algorithms to divide fractions.
  - Multiplying by the denominator and dividing by the numerator
  - Multiplying by the reciprocal
  - Common denominator approach
- find missing values in a number sentence.

RESOURCES
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STATE STANDARD

4.1.6A.1 Use real-life experiences, physical materials, and technology to construct meanings for numbers
- All integers
- All fractions as part of a whole, as subset of a set, as a location on a number line, and as divisions of whole numbers
- All decimals

4.1.6B.4 Select pencil-and-paper, mental math, or a calculator as the appropriate computational method in a given situation depending on the context and numbers.

4.1.6B.6 Check the reasonableness of results of computations.

4.1.6B.8 Understand and apply the standard algebraic order of operations for the four basic operations, including appropriate use of parentheses.

4.2.6B.2 Recognize, identify, and describe geometric relationships and properties as they exist in nature, art, and other real-world settings.

4.2.6D.1 Select and use appropriate units to measure angles, area, surface area, and volume.

4.2.6D.2 Use a scale to find a distance on a map or a length on a scale drawing.

4.2.6D.5 Use measurements and estimates to describe and compare phenomena.

4.2.6E.1 Use a protractor to measure angles.

4.2.6E.2 Develop and apply strategies and formulas for finding perimeter and area.
- Triangle, square, rectangle, parallelogram, and trapezoid
- Circumference and area of a circle

4.2.6E.4 Recognize that shapes with the same perimeter do not necessarily have the same area and vice versa.

4.2.6E.5 Develop informal ways of approximating the measures of familiar objects (e.g., use a grid to approximate the area of the bottom of one’s foot).

4.3.6C.1 Use patterns, relations, and linear functions to model situations.
- Using variables to represent unknown quantities
- Using concrete materials, tables, graphs, verbal rules, algebraic expressions/equations/inequalities

4.5A.1 Learn mathematics through problem solving, inquiry, and discovery.

4.5A.2 Solve problems that arise in mathematics and in other contexts.
- Open-ended problems
- Non-routine problems
- Problems with multiple solutions
- Problems that can be solved in several ways
4.5A.3 Select and apply a variety of appropriate problem-solving strategies (e.g., “try a simpler problem” or “make a diagram”) to solve problems.
4.5A.4 Pose problems of various types and levels of difficulty.
4.5A.5 Monitor their progress and reflect on the process of their problem solving activity.
4.5B.1 Use communication to organize and clarify mathematical thinking.
   - Reading and writing
   - Discussion, listening, and questioning
4.5B.2 Communicate mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
4.5B.3 Analyze and evaluate the mathematical thinking and strategies of others.
4.5B.4 Use the language of mathematics to express mathematical ideas precisely.
4.5C.1 Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
4.5C.2 Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
4.5C.3 Recognize that mathematics is used in a variety of contexts outside of mathematics.
4.5C.4 Apply mathematics in practical situations and in other disciplines.
4.5C.5 Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
4.5C.6 Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
4.5D.1 Recognize that mathematical facts, procedures, and claims must be justified.
4.5D.2 Use reasoning to support their mathematical conclusions and problem solutions.
4.5D.3 Select and use various types of reasoning and methods of proof.
4.5D.4 Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
4.5D.5 Make and investigate mathematical conjectures.
   - Counterexamples as a means of disproving conjectures.
   - Verifying conjectures using informal reasoning or proofs.
4.5D.6 Evaluate examples of mathematical reasoning and determine whether they are valid.
4.5E.1 Create and use representations to organize, record, and communicate mathematical ideas.
   - Concrete representations (e.g., base-ten blocks or algebra tiles)
   - Pictorial representations (e.g., diagrams, charts, or tables)
   - Symbolic representations (e.g., a formula)
   - Graphical representations (e.g., a line graph)
4.5E.2 Select, apply, and translate among mathematical representations to solve problems.
4.5E.3 Use representations to model and interpret physical, social, and mathematical phenomena.
4.5F.1 Use technology to gather, analyze, and communicate mathematical information.
4.5F.2 Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.
4.5F.3 Use graphing calculators and computer software to investigate properties of functions and their graphs.
4.5F.4 Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
4.5F.5 Use computer software to make and verify conjectures about geometric objects.
4.5F.6 Use computer-based laboratory technology for mathematical applications in the sciences.

BIG IDEAS/COMMON THREADS
All students will understand the meaning of numbers, how they may be represented and the relationships among them. They will perform computations and acquire knowledge of the physical world from the point of view of quantitative relationships.

ENDURING UNDERSTANDINGS
Strategies can be developed and applied to measure specific attributes of geometric figures.

ESSENTIAL QUESTIONS
PRIMARY: Why is two-dimensional measurement necessary to solve problems?
SECONDARY: How do you find the area and/or perimeter of figures? What is the relationship between area and perimeter of a figure?

MODULE ASSESSMENT
Unit 5 Assessment

LESSON OBJECTIVES
Students will be able to...

• use area and relate areas to covering a figure.
• use perimeter and relate perimeter to surrounding a figure.
• Analyze what it means to measure area and perimeter.
• develop strategies for finding areas and perimeters of rectangular and non-rectangular shapes.
• discover relationships between perimeter and area, including that one can vary while the other stays fixed.
• analyze how the area of a triangle and the area of a parallelogram are related to the area of a rectangle.
• develop formulas and procedures, stated in words or symbols, for finding areas and perimeters of rectangles, parallelograms, triangles, and circles.
• develop techniques for estimating the area and perimeter of an irregular figure.
• recognize situations in which measuring perimeter or area will help answer practical questions.

MODULE SKILLS
Students will be able to...
• identify units of measurement for problems involving area and/or perimeter.
• find the area and perimeter of a figure by counting, given that the figure is presented with square tiles, grid paper, or grids.
• find the area of a rectangle, given a diagram or the dimensions of the rectangle.
  ▪ Counting tiles on a grid
  ▪ Using the algorithm of length times width
• find the perimeter of a rectangle, given a diagram or the dimensions of the rectangle.
  ▪ Counting number of squares along the sides on a grid
  ▪ Using an algorithm
    • Perimeter = (length + width) x 2
    • Perimeter = 2length + 2width
    • Perimeter = length + width + length + width
• find the area of a triangle.
  ▪ By surrounding a triangle with a rectangle, then taking half of the rectangle’s area.
  ▪ Using an algorithm
    • Area = ½ (length x width)
• find the perimeter of a triangle.
• find the area of a parallelogram, given a diagram or the appropriate dimensions of the parallelogram.
  ▪ Rearrange the parallelogram as a rectangle.
  ▪ Using an algorithm
    • Area = base x height
• find the area and circumference of a circle.
  ▪ Using an algorithm
    • Area = \( \pi r^2 \)
    • Perimeter = \( 2\pi r = \pi d \)
• estimate the area and perimeter of any figure, recognizing that the some scales are more precise estimates than others.
• find possible dimensions and perimeter of a figure, given a fixed area. Students must recognize that there are many possibilities.
• find possible dimensions and area of a figure, given a fixed perimeter. Students must recognize that there are many possibilities.

RESOURCES
Page 36
STATE STANDARD
4.1.6A.1 Use real-life experiences, physical materials, and technology to construct meanings for numbers
- All integers
- All fractions as part of a whole, as subset of a set, as a location on a number line, and as divisions of whole numbers
- All decimals
4.1.6A.2 Recognize the decimal nature of United States currency and compute with money.
4.1.6A.3 Demonstrate a sense of the relative magnitudes of numbers.
4.1.6A.4 Explore the use of ratios and proportions in a variety of situations.
4.1.6A.5 Understand and use whole-number percents between 1 and 100 in a variety of situations.
4.1.6A.6 Use whole numbers, fractions, and decimals to represent equivalent forms of the same number.
4.1.6A.8 Compare and order numbers.
4.1.6B.1 Recognize the appropriate use of each arithmetic operation in problem situations.
4.1.6B.2 Construct, use, and explain procedures for performing calculations with fractions and decimals with:
- Pencil-and-paper
- Mental math
- Calculator
4.1.6B.3 Use an efficient and accurate pencil-and-paper procedure for division of a 3-digit number by a 2-digit number.
4.1.6B.4 Select pencil-and-paper, mental math, or a calculator as the appropriate computational method in a given situation depending on the context and numbers.
4.1.6B.6 Check the reasonableness of results of computations.
4.1.6B.7 Understand and use the various relationships among operations and properties of operations.
4.1.6B.8 Understand and apply the standard algebraic order of operations for the four basic operations, including appropriate use of parentheses.
4.1.6C.1 Use a variety of strategies for estimating both quantities and the results of computations.
4.1.6C.2 Recognize when an estimate is appropriate, and understand the usefulness of an estimate as distinct from an exact answer.
4.1.6C.3 Determine the reasonableness of an answer by estimating the result of operations.
4.1.6C.4 Determine whether a given estimate is an overestimate or an underestimate.

4.3.6C.1 Use patterns, relations, and linear functions to model situations.
  - Using variables to represent unknown quantities
  - Using concrete materials, tables, graphs, verbal rules, algebraic expressions/equations/inequalities

4.3.6D.2 Understand and apply the properties of operations and numbers.
  - Distributive property
  - The product of a number and its reciprocal is 1

4.3.6D.3 Evaluate numerical expressions.

4.5A.1 Learn mathematics through problem solving, inquiry, and discovery.

4.5A.2 Solve problems that arise in mathematics and in other contexts.
  - Open-ended problems
  - Non-routine problems
  - Problems with multiple solutions
  - Problems that can be solved in several ways

4.5A.3 Select and apply a variety of appropriate problem-solving strategies (e.g., “try a simpler problem” or “make a diagram”) to solve problems.

4.5A.4 Pose problems of various types and levels of difficulty.

4.5A.5 Monitor their progress and reflect on the process of their problem solving activity.

4.5B.1 Use communication to organize and clarify mathematical thinking.
  - Reading and writing
  - Discussion, listening, and questioning

4.5B.2 Communicate mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.

4.5B.3 Analyze and evaluate the mathematical thinking and strategies of others.

4.5B.4 Use the language of mathematics to express mathematical ideas precisely.

4.5C.1 Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).

4.5C.2 Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).

4.5C.3 Recognize that mathematics is used in a variety of contexts outside of mathematics.

4.5C.4 Apply mathematics in practical situations and in other disciplines.

4.5C.5 Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).

4.5C.6 Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

4.5D.1 Recognize that mathematical facts, procedures, and claims must be justified.

4.5D.2 Use reasoning to support their mathematical conclusions and problem solutions.

4.5D.3 Select and use various types of reasoning and methods of proof.
4.5D.4 Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
4.5D.5 Make and investigate mathematical conjectures.
   - Counterexamples as a means of disproving conjectures.
   - Verifying conjectures using informal reasoning or proofs.
4.5D.6 Evaluate examples of mathematical reasoning and determine whether they are valid.
4.5E.1 Create and use representations to organize, record, and communicate mathematical ideas.
   - Concrete representations (e.g., base-ten blocks or algebra tiles)
   - Pictorial representations (e.g., diagrams, charts, or tables)
   - Symbolic representations (e.g., a formula)
   - Graphical representations (e.g., a line graph)
4.5E.2 Select, apply, and translate among mathematical representations to solve problems.
4.5E.3 Use representations to model and interpret physical, social, and mathematical phenomena.
4.5F.1 Use technology to gather, analyze, and communicate mathematical information.
4.5F.2 Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.
4.5F.3 Use graphing calculators and computer software to investigate properties of functions and their graphs.
4.5F.4 Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
4.5F.5 Use computer software to make and verify conjectures about geometric objects.
4.5F.6 Use computer-based laboratory technology for mathematical applications in the sciences.

BIG IDEAS/COMMON THREADS
  All students will understand the meaning of numbers, how they may be represented and the relationships among them. They will perform computations and acquire knowledge of the physical world from the point of view of quantitative relationships.

ENDURING UNDERSTANDINGS
  Decimals and percents are utilized in our world on a daily basis, with calculations and operations on decimals and percents as a vital element.

ESSENTIAL QUESTIONS
  PRIMARY: Why do we need to perform operations involving decimals and percents?
  SECONDARY: What algorithms and visual models can help to perform these operations?

MODULE ASSESSMENT
  Unit 6 Assessment
LESSON OBJECTIVES
Students will be able to...
• use your knowledge of fractions to learn about operating with decimals.
• consider the relative size of a decimal prior to developing approaches to finding exact decimal sums or differences.
• develop place-value understanding of decimal addition and subtraction.
• connect strategies for addition and subtraction of decimals to additional and subtraction of fractions with powers of ten in the denominator.
• estimate the results of operations with decimals.
• relate renaming fractions to have a common denominator to the notion of adding values with the same place value.
• consider how finding a decimal part of and a fraction part of a quantity affects the relative size of the product.
• understand what happens to place value and the position of the decimal when you multiply by powers of ten.
• know when to use each operation in a situation involving decimals.
• Understand and predict the decimal representation of a fraction (terminating or repeating).
• understand that a percent is a decimal fraction with a denominator of 100.
• recognize real-world situations where people often choose to use decimals instead of common fractions.
• develop algorithms for solving a variety of types of percent problems.

MODULE SKILLS
Students will be able to...
• use benchmark and decimal-fraction relationships to develop estimation strategies for finding decimal sums.
• use algorithms to add and subtract decimals.
• solve problems that require decimal addition and subtraction.
• estimate the relative size of a decimal product prior to finding an exact answer.
• use algorithms to multiply decimals.
• solve problems that require decimal multiplication.
• use estimation as a strategy for locating the position of the decimal in finding exact decimal products.
• use models and the context to find solutions to division problems.
• use efficient algorithms for dividing decimals.
• represent percents as decimals and use decimal computation to compute percents.
• use percents in estimating or computing taxes, tips, and discounts.
• solve problems using percents.

RESOURCES
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Oradell, River Dell and River Edge Public School Districts
Mathematics Curriculum – Sixth Grade
Approved June/July 2009
RIVER EDGE MATHEMATICS CURRICULUM
GRADE SIX
UNDERSTANDING PROBABILITY

STATE STANDARD
4.1.6A.1 Use real-life experiences, physical materials, and technology to construct meanings for numbers
- All integers
- All fractions as part of a whole, as subset of a set, as a location on a number line, and as divisions of whole numbers
- All decimals
4.1.6A.5 Understand and use whole-number percents between 1 and 100 in a variety of situations.
4.3.6C.1 Use patterns, relations, and linear functions to model situations.
- Using variables to represent unknown quantities
- Using concrete materials, tables, graphs, verbal rules, algebraic expressions/equations/inequalities
4.4.6B.1 Determine probabilities of events.
- Event, complementary event, probability of an event
- Multiplication rule for probabilities
- Probability of certain event is 1 and of impossible event is 0
- Probabilities of event and complementary event add up to 1
4.4.6B.2 Determine probability using intuitive, experimental, and theoretical methods (e.g., using model of picking items of different colors from a bag).
- Given numbers of various types of items in a bag, what is the probability that an item of one type will be picked
- Given data obtained experimentally, what is the likely distribution of items in the bag
4.4.6B.3 Explore compound events.
4.4.6B.4 Model situations involving probability using simulations (with spinners, dice) and theoretical models.
4.4.6B.5 Recognize and understand the connections among the concepts of independent outcomes, picking at random, and fairness.
4.4.6C.1 Solve counting problems and justify that all possibilities have been enumerated without duplication.
- Organized lists, charts, tree diagrams, tables
- Venn diagrams
4.4.6C.2 Apply the multiplication principle of counting.
- Simple situations (e.g., you can make $3 \times 4 = 12$ outfits using 3 shirts and 4 skirts).
- Number of ways a specified number of items can be arranged in order (concept of permutation)
- Number of ways of selecting a slate of officers from a class (e.g., if there are 23 students and 3 officers, the number is 23 x 22 x 21)

4.4.6C.3 List the possible combinations of two elements chosen from a given set (e.g., forming a committee of two from a group of 12 students, finding how many handshakes there will be among ten people if everyone shakes each other person's hand once).

4.5A.1 Learn mathematics through problem solving, inquiry, and discovery.
4.5A.2 Solve problems that arise in mathematics and in other contexts.
  - Open-ended problems
  - Non-routine problems
  - Problems with multiple solutions
  - Problems that can be solved in several ways

4.5A.3 Select and apply a variety of appropriate problem-solving strategies (e.g., “try a simpler problem” or “make a diagram”) to solve problems.
4.5A.4 Pose problems of various types and levels of difficulty.
4.5A.5 Monitor their progress and reflect on the process of their problem solving activity.

4.5B.1 Use communication to organize and clarify mathematical thinking.
  - Reading and writing
  - Discussion, listening, and questioning

4.5B.2 Communicate mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
4.5B.3 Analyze and evaluate the mathematical thinking and strategies of others.
4.5B.4 Use the language of mathematics to express mathematical ideas precisely.

4.5C.1 Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
4.5C.2 Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
4.5C.3 Recognize that mathematics is used in a variety of contexts outside of mathematics.
4.5C.4 Apply mathematics in practical situations and in other disciplines.
4.5C.5 Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
4.5C.6 Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

4.5D.1 Recognize that mathematical facts, procedures, and claims must be justified.
4.5D.2 Use reasoning to support their mathematical conclusions and problem solutions.
4.5D.3 Select and use various types of reasoning and methods of proof.
4.5D.4 Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
4.5D.5 Make and investigate mathematical conjectures.
  - Counterexamples as a means of disproving conjectures.
• Verifying conjectures using informal reasoning or proofs.
4.5D.6 Evaluate examples of mathematical reasoning and determine whether they are valid.
4.5E.1 Create and use representations to organize, record, and communicate mathematical ideas.
  • Concrete representations (e.g., base-ten blocks or algebra tiles)
  • Pictorial representations (e.g., diagrams, charts, or tables)
  • Symbolic representations (e.g., a formula)
  • Graphical representations (e.g., a line graph)
4.5E.2 Select, apply, and translate among mathematical representations to solve problems.
4.5E.3 Use representations to model and interpret physical, social, and mathematical phenomena.
4.5F.1 Use technology to gather, analyze, and communicate mathematical information.
4.5F.2 Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.
4.5F.3 Use graphing calculators and computer software to investigate properties of functions and their graphs.
4.5F.4 Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
4.5F.5 Use computer software to make and verify conjectures about geometric objects.
4.5F.6 Use computer-based laboratory technology for mathematical applications in the sciences.

BIG IDEAS/COMMON THREADS
All students will understand the meaning of numbers, how they may be represented and the relationships among them. They will perform computations and acquire knowledge of the physical world from the point of view of quantitative relationships.

ENDURING UNDERSTANDINGS
Theoretical and experimental probabilities can be used to predict future events.

ESSENTIAL QUESTIONS
PRIMARY: How can probability assist in making predictions for future events?
SECONDARY: How are theoretical and experimental probabilities calculated? How can possible event outcomes be displayed visually?

MODULE ASSESSMENT
Unit 7 Assessment
LESSON OBJECTIVES
Students will be able to...

- use probabilities to predict what will happen over the long run.
- use the concepts of equally likely and not equally likely.
- understand that a game of chance is fair only if each player has the same chance of winning, not just a possible chance of winning.
- understand that there are two ways to build probability models: by gathering data from experiments (experimental probability) and by analyzing possible equally likely outcomes (theoretical probability).
- understand that experimental probabilities are better estimates of theoretical probabilities when they are based on larger numbers of trials.
- develop strategies for finding both experimental and theoretical probabilities.
- interpret statements of probability to make decisions and answer questions.

MODULE SKILLS
Students will be able to...

- create an experiment designed to prevent bias of the outcome (fair trial, systematic bias, random data collection).
- state the probability of an event based on the Law of Large Numbers.
- use the results of an experiment to show the relative frequency of an event occurring.
- find experimental probabilities for situations involving different consecutive events.
- find theoretical probabilities for situations involving different consecutive events.
- use probability notation to express the probability of an event(s).
- create a tree diagram or organized list to systematically determine all possible outcomes of a situation.
- determine if possible results are equally likely or not equally likely.
- find the probability of compound events ("and", "or").
- describe the difference between probable and possible events.

RESOURCES
Page 36
STATE STANDARD
4.1.6A.1 Use real-life experiences, physical materials, and technology to construct meanings for numbers
- All integers
- All fractions as part of a whole, as subset of a set, as a location on a number line, and as divisions of whole numbers
- All decimals
4.1.6A.4 Explore the use of ratios and proportions in a variety of situations.
4.1.6B.4 Select pencil-and-paper, mental math, or a calculator as the appropriate computational method in a given situation depending on the context and numbers.
4.3.6C.1 Use patterns, relations, and linear functions to model situations.
- Using variables to represent unknown quantities
- Using concrete materials, tables, graphs, verbal rules, algebraic expressions/equations/inequalities
4.4.6A.1 Collect, generate, organize, and display data.
- Data generated from surveys
4.4.6A.2 Read, interpret, select, construct, analyze, generate questions about, and draw inferences from displays of data.
- Bar graph, line graph, circle graph, table, histogram
- Range, median, and mean
- Calculators and computers used to record and process information
4.4.6A.3 Respond to questions about data, generate their own questions and hypotheses, and formulate strategies for answering their questions and testing their hypotheses.
4.5A.1 Learn mathematics through problem solving, inquiry, and discovery.
4.5A.2 Solve problems that arise in mathematics and in other contexts.
- Open-ended problems
- Non-routine problems
- Problems with multiple solutions
- Problems that can be solved in several ways
4.5A.3 Select and apply a variety of appropriate problem-solving strategies (e.g., “try a simpler problem” or “make a diagram”) to solve problems.
4.5A.4 Pose problems of various types and levels of difficulty.
4.5A.5 Monitor their progress and reflect on the process of their problem solving activity.
4.5B.1 Use communication to organize and clarify mathematical thinking.
- Reading and writing
- Discussion, listening, and questioning
4.5B.2 Communicate mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.
4.5B.3 Analyze and evaluate the mathematical thinking and strategies of others.
4.5B.4 Use the language of mathematics to express mathematical ideas precisely.
4.5C.1 Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
4.5C.2 Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
4.5C.3 Recognize that mathematics is used in a variety of contexts outside of mathematics.
4.5C.4 Apply mathematics in practical situations and in other disciplines.
4.5C.5 Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
4.5C.6 Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
4.5D.1 Recognize that mathematical facts, procedures, and claims must be justified.
4.5D.2 Use reasoning to support their mathematical conclusions and problem solutions.
4.5D.3 Select and use various types of reasoning and methods of proof.
4.5D.4 Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
4.5D.5 Make and investigate mathematical conjectures.
   - Counterexamples as a means of disproving conjectures.
   - Verifying conjectures using informal reasoning or proofs.
4.5D.6 Evaluate examples of mathematical reasoning and determine whether they are valid.
4.5E.1 Create and use representations to organize, record, and communicate mathematical ideas.
   - Concrete representations (e.g., base-ten blocks or algebra tiles)
   - Pictorial representations (e.g., diagrams, charts, or tables)
   - Symbolic representations (e.g., a formula)
   - Graphical representations (e.g., a line graph)
4.5E.2 Select, apply, and translate among mathematical representations to solve problems.
4.5E.3 Use representations to model and interpret physical, social, and mathematical phenomena.
4.5F.1 Use technology to gather, analyze, and communicate mathematical information.
4.5F.2 Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.
4.5F.3 Use graphing calculators and computer software to investigate properties of functions and their graphs.
4.5F.4 Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
4.5F.5 Use computer software to make and verify conjectures about geometric objects.
4.5F.6 Use computer-based laboratory technology for mathematical applications in the sciences.

BIG IDEAS/COMMON THREADS
All students will understand the meaning of numbers, how they may be represented and the relationships among them. They will perform computations and acquire knowledge of the physical world from the point of view of quantitative relationships.

ENDURING UNDERSTANDINGS
Mathematical data can be collected, organized, described, analyzed, and displayed, in order to help make decisions and/or predictions.

ESSENTIAL QUESTIONS
PRIMARY: What does the process of data investigation consist of?
SECONDARY: How are statistics collected, analyzed, interpreted, and displayed?

MODULE ASSESSMENT
Unit 8 Assessment

LESSON OBJECTIVES
Students will be able to...

• understand and use the process of data investigation: posing questions, collecting and analyzing data distributions, and making interpretations to answer questions.
• represent distributions of data using line plots, bar graphs, stem-and-leaf plots, and coordinate graphs.
• compute the mean, median, mode, and range of the data.
• distinguish between categorical data and numerical data and identify which graphs and statistics may be used to represent each kind of data.
• make informed decisions about which graph(s) and which of the measures of center (mean, median, or mode) and range may be used to describe a distribution of data.
• develop strategies for comparing distributions of data.

MODULE SKILLS
Students will be able to...

• collect data and organize it visually.
• find the mean, median, mode, and range of a numerical data set.
• find the mode of a categorical data set.
• create a frequency chart for numerical or categorical data.
• represent distributions of data using line plots, bar graphs, stem-and-leaf plots, and coordinate graphs.
• describe data distributions.
• compare two distributions displayed using back-to-back stem-and-leaf plots.
• compare two distributions using statistics, such as median, range, and how the data vary from least to greatest values.
• identify outliers in a distribution.
• locate information from a graph to answer explicit questions.
• use clusters of information presented in a graph to ‘read between the data.’
• use data to extend, predict, or infer to answer questions that go ‘beyond the data.’
• use information about values from one variable to help understand and explain or predict values of another variable. In other words, decide if changes in one variable are related to changes in the other variable.
• recognize that data with the same mean may have different distributions.

RESOURCES
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RESOURCES

FACTORS AND MULTIPLES
*Supplies:* Student notebooks, calculators, blank transparencies, transparency markers, colored pencils, paper clips, colored chips, square tiles, grid paper, scissors, tape, two-color chips, student workbooks, and CMP2 teacher resources.
*Texts:* Connected Mathematics Project 2
Glencoe Math Textbook
*Websites:* [http://connectedmath.msu.edu/](http://connectedmath.msu.edu/)  

UNDERSTANDING FRACTIONS, DECIMALS, AND PERCENTS
*Supplies:* Student notebooks, calculators, blank transparencies, transparency markers, 8½ inch strips of paper, scissors, index cards, Post-It Notes, student workbooks, and CMP2 teacher resources.
*Texts:* Connected Mathematics Project 2
Glencoe Math Textbook
*Websites:* [http://connectedmath.msu.edu/](http://connectedmath.msu.edu/)  

TWO-DIMENSIONAL GEOMETRY
*Supplies:* Student notebooks, calculators, blank transparencies, transparency markers, shapes sets, poster paper, angle rulers or protractors, construction paper, scissors, polystrips, fasteners, number cubes, rubber bands, geoboards, rulers, student workbooks, and CMP2 teacher resources.
*Texts:* Connected Mathematics Project 2
Glencoe Math Textbook
*Websites:* [http://connectedmath.msu.edu/](http://connectedmath.msu.edu/)  

USING FRACTION OPERATIONS
*Supplies:* Student notebooks, calculators, blank transparencies, transparency markers, scissors, chart paper, colored pencils, student workbooks, and CMP2 teacher resources.
*Texts:* Connected Mathematics Project 2
Glencoe Math Textbook
*Websites:* [http://connectedmath.msu.edu/](http://connectedmath.msu.edu/)  

TWO-DIMENSIONAL MEASUREMENTS
*Supplies:* Student notebooks, calculators, blank transparencies, transparency markers, square tiles, grid paper, string, scissors, rulers, tape measures, circular objects of different sizes, glue, construction paper, student workbooks, and CMP2 teacher resources.
*Texts:* Connected Mathematics Project 2
Glencoe Math Textbook

Websites: [http://connectedmath.msu.edu/](http://connectedmath.msu.edu/)  

**COMPUTING WITH DECIMALS AND PERCENTS**

*Supplies:* Student notebooks, calculators, blank transparencies, transparency markers, protractors, angle rulers, student workbooks, and CMP2 teacher resources.

*Texts:* Connected Mathematics Project 2  
Glencoe Math Textbook

*Websites:* [http://connectedmath.msu.edu/](http://connectedmath.msu.edu/)  

**UNDERSTANDING PROBABILITY**

*Supplies:* Student notebooks, calculators, blank transparencies, transparency markers, coins, paper cups, opaque containers, colored blocks, chart paper, paper clips, angle rulers, number cubes, bags, game markers, student workbooks, and CMP2 teacher resources.

*Texts:* Connected Mathematics Project 2  
Glencoe Math Textbook

*Websites:* [http://connectedmath.msu.edu/](http://connectedmath.msu.edu/)  

**STATISTICS**

*Supplies:* Student notebooks, calculators, blank transparencies, transparency markers, class list of students, stick-on notes, centimeter grid paper, graph paper, scissors, index cards, yardsticks, string, cubes, unlined paper, colored stick on dots, local street map, gridded chart paper, student workbooks, and CMP2 teacher resources.

*Texts:* Connected Mathematics Project 2  
Glencoe Math Textbook

*Websites:* [http://connectedmath.msu.edu/](http://connectedmath.msu.edu/)  